

Modeling Nonlinear Behavior of Band-Pass Memoryless and Dynamic Systems

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Abstract – This paper addresses nonlinear distortion arising in microwave band-pass memoryless and dynamic systems. It first identifies the minimum requirements for their correct representation. Then, it shows that the complex behavior of long term memory effects does not allow a unique characterization procedure, but demands for various nonlinear distortion figures according to the type of nonlinear RF impairments the actual system is sensitive to.

I. INTRODUCTION

The analysis of band-pass memoryless systems has deserved a strong attention for now more than thirty years [1]. But, recently, the microwave community began to realize that such approximation was insufficient to accurately design wideband power amplifier, PA, linearizers. Consequently, a growing attention has been paid to nonlinear distortion effects arising from band-pass systems showing significant long term memory, and to their accurate modeling [2,3].

The main purpose of this paper is to present the modeling requirements of those microwave band-pass memoryless and dynamic systems, taking into consideration their general nonlinear distortion behavior. Then, those results are used in the discussion of the appropriateness of most important nonlinear distortion figures of merit and their corresponding laboratory measurement set-ups.

II. BAND-PASS MEMORYLESS AND DYNAMIC SYSTEMS' REPRESENTATION

Microwave and wireless PAs may present memory effects that have short and long time-constants compared to the RF carrier signal or to its slowly-varying envelope.

For the short memory effects contribute the band-pass characteristics of the PA input and output matching networks and, sometimes, also the low-pass characteristics of the active device. These can be modeled by two filters with a memoryless transfer nonlinearity in between.

Modeling long term nonlinear memory effects is much more difficult. Theoretical and experimental works have related those effects to a large variety of PA characteristics that span from low frequency dispersion induced by long

time-constant traps and thermal constants, deliberate or accidental envelope feedback and long time-constants present in the input and output bias circuitry. Apparently, it seems that, from these, bias circuitry induced memory is, with a more or less extent, common to almost all PA circuits, being particularly important in the output of FET based PAs, and in the input of bipolar transistor ones [2]. In any case, it should be obvious that in usual band-pass microwave PAs intended to handle signals that occupy only a small percentage of their available bandwidth (this way leading to instantaneous responses), there must be some low frequency, LF, component (the envelope), for which the circuit is no longer memoryless, that must be remixed with the original RF signal to create those long term memory effects. But, since these LF envelope components can only be generated (or demodulated from the RF signal) in a nonlinearity, no transfer nonlinearity can remix them again with the original signal to produce new in-band intermodulation products, unless some form of LF feedback is available. Note, however, that this LF feedback needs not to be a physical path, but simply the conceptual feedback present whenever, e.g., the output current of a FET, at the envelope frequencies, generated from the $i_{DS}(v_{GS})$ nonlinearity, circulates in the load impedance mesh and is converted into a $V_{ds}(a_{LF})$ that is then remixed with the original $V_{ds}(a_{RF})$ signal in the $i_{DS}(v_{DS})$ nonlinearities.

Beyond this LF feedback, remixing even order high-frequency, HF, components with the original ones, can also produce in-band intermodulation components. Therefore, memory pressed into these HF components can also be a cause of band-pass dynamic behavior.

This is illustrated in the PA simplified circuit of Fig. 1 and the corresponding system model of Fig. 2.

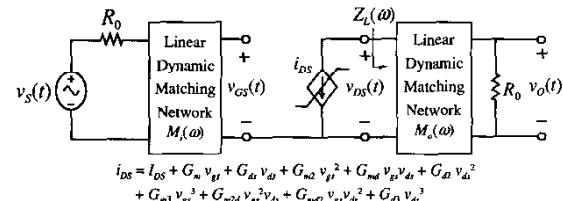


Fig. 1 Simplified FET based PA circuit used for the nonlinear analysis.

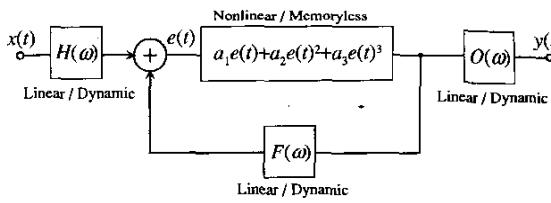


Fig. 2 Nonlinear band-pass dynamic system model of the PA circuit represented in Fig. 1.

Even if the PA circuit of Fig. 1 is unilateral, the conceptual feedback above described is indeed still present. This may be easily concluded by comparing the Volterra nonlinear transfer functions, NLTFs, obtained from this circuit and the ones of the system model of Fig. 2.

Actually, using a straightforward mildly nonlinear Volterra series analysis of both the unilateral PA circuit and the general system model, it can be shown that their band-pass characteristics (either signal or distortion) are represented by dynamic behavioral models (i.e the first few odd-order Volterra frequency-domain NLTFs, $H_n(\omega_1, \dots, \omega_n)$, where $n=1,3$) that are formally identical.

Indeed, we have for the PA's linear transfer function (in which the input and output are $v_S(t)$ and $v_O(t)$, respectively):

$$S_1(\omega) = -M_i(\omega)G_m \frac{Z_L(\omega)}{D'(\omega)} M_o(\omega) \quad (1)$$

where $D'(\omega) = 1 + G_{ds}Z_L(\omega)$. 3rd order NLTF is:

$$\begin{aligned} S_3(\omega_1, \omega_2, \omega_3) &= -M_i(\omega_1)M_i(\omega_2)M_i(\omega_3) \\ &\quad \frac{Z_L(\omega_1 + \omega_2 + \omega_3)}{D'(\omega_1 + \omega_2 + \omega_3)} M_o(\omega_1 + \omega_2 + \omega_3) \\ &\quad \left\{ \left(G_{m3} + G_{m2d}A_v + G_{md2}A_v^2 + G_{d3}A_v^3 \right) \right. \\ &\quad - \frac{1}{3} (G_{md} + 2G_{d2}A_v) \left(G_{m2} + G_{md}A_v + G_{d2}A_v^2 \right) \\ &\quad \left. \left[\frac{Z_L(\omega_1 + \omega_2)}{D'(\omega_1 + \omega_2)} + \frac{Z_L(\omega_1 + \omega_3)}{D'(\omega_1 + \omega_3)} + \frac{Z_L(\omega_2 + \omega_3)}{D'(\omega_2 + \omega_3)} \right] \right\} \end{aligned} \quad (2)$$

where, under the band-pass approximation, it is assumed that $A_v(\omega) = -G_m \frac{Z_L(\omega)}{D'(\omega)}$ is constant within the signal

bandwidth and equals A_v or A_v^* , whether ω is a positive or negative input frequency quantity.

On the other hand, for the system model we have:

$$S_1(\omega) = H(\omega) \frac{a_1}{D(\omega)} O(\omega) \quad (3)$$

and

$$\begin{aligned} S_3(\omega_1, \omega_2, \omega_3) &= \frac{H(\omega_1)H(\omega_2)H(\omega_3)}{D(\omega_1)D(\omega_2)D(\omega_3)} \frac{O(\omega_1 + \omega_2 + \omega_3)}{D(\omega_1 + \omega_2 + \omega_3)} \\ &\quad \left\{ a_3 + \frac{2}{3} a_2^2 \left[\frac{F(\omega_1 + \omega_2)}{D(\omega_1 + \omega_2)} + \frac{F(\omega_1 + \omega_3)}{D(\omega_1 + \omega_3)} + \frac{F(\omega_2 + \omega_3)}{D(\omega_2 + \omega_3)} \right] \right\} \end{aligned} \quad (4)$$

where $D(\omega) = 1 - a_1 F(\omega)$.

So, if now a general Q -tone stimulus is assumed,

$$x(t) = \frac{1}{2} \sum_{q=-Q}^Q X_q e^{j\omega_q t} \quad (5)$$

the in-band PA output can be given by:

$$\begin{aligned} y(t) &= \frac{1}{2} \sum_{q=-Q}^Q X_q S_1(\omega_q) e^{j\omega_q t} \\ &\quad + \frac{1}{2^3} \sum_{q_1=-Q}^Q \sum_{q_2=-Q}^Q \sum_{q_3=-Q}^Q X_{q_1} X_{q_2} X_{q_3} \\ &\quad S_3(\omega_{q_1}, \omega_{q_2}, \omega_{q_3}) e^{j(\omega_{q_1} + \omega_{q_2} + \omega_{q_3})t} \end{aligned} \quad (6)$$

in which ω_{q_1} , ω_{q_2} and ω_{q_3} can be any two positive and one negative input frequencies.

This output includes linear terms at the fundamental signal components ω_q :

$$y_L(t) = \frac{1}{2} \sum_{q=-Q}^Q X_q S_1(\omega_q) e^{j\omega_q t} \quad (7)$$

nonlinear distortion terms also at these fundamentals:

$$\begin{aligned} y_N(t) &= \\ &\quad \frac{6}{2^3} \sum_{q_1=-Q}^Q \sum_{q_2=-Q}^Q \sum_{\substack{q_3=-Q \\ q_2 \neq q_1}}^Q X_{q_1} |X_{q_2}|^2 S_3(\omega_{q_1}, \omega_{q_2}, -\omega_{q_2}) e^{j\omega_{q_1} t} \\ &\quad + \frac{3}{2^3} \sum_{q_1=-Q}^Q X_{q_1} |X_{q_1}|^2 S_3(\omega_{q_1}, \omega_{q_1}, -\omega_{q_1}) e^{j\omega_{q_1} t} \end{aligned} \quad (8)$$

and, finally, nonlinear distortion spectral regrowth:

$$\begin{aligned} y_S(t) &= \frac{1}{2^3} \sum_{q_1=-Q}^Q \sum_{q_2=-Q}^Q \sum_{q_3=-Q}^Q X_{q_1} X_{q_2} X_{q_3} \\ &\quad S_3(\omega_{q_1}, \omega_{q_2}, \omega_{q_3}) e^{j(\omega_{q_1} + \omega_{q_2} + \omega_{q_3})t} \end{aligned} \quad (9)$$

where ω_{q_1} , ω_{q_2} and ω_{q_3} can be any two positive and one negative input frequencies, but in which the negative one can not be the symmetric of any of the other two.

Assuming a large number of tones of equal amplitude and random phases (a used approximation of band-limited white Gaussian noise), it can be shown that, while $y_L(t)$

and $y_N(t)$ are correlated with the input signal $x(t)$, $y_S(t)$ is not, therefore behaving as a stochastic perturbation.

Noting that the fundamental signal output is given by (7) and (8), and considering the form of (4), it can be easily concluded that it is the eventual presence of $F(\omega_{HF})$ (where $\omega_{HF}=2\omega_1$ or $\omega_{HF}=\omega_1+\omega_2$) or a reactive component of $F(\omega_{HF})$ that gives a phase of $S_3(\omega, \omega, \omega)$ different from the one of $S_1(\omega)$. These are thus responsible for describing PA AM-PM conversion. Or, referring to the unilateral PA circuit of Fig. 1, it is the reactive behavior of $Z_L(\omega_{HF})$ and the 2nd degree remixing at G_{md} and G_{d2} , or a reactive component of $Z_L(\omega_{HF})$, that are responsible for that band-pass memoryless effect.

Furthermore, it can be also concluded that only the presence of non-null low frequency or high frequency feedback, $F(\omega_{LF})$, $F(\omega_{HF})$ (where $\omega_{LF,HF}=\omega_1 \pm \omega_2$, $\omega_{LF,HF}=\omega_1 \pm \omega_3$ or $\omega_{LF,HF}=\omega_2 \pm \omega_3$), can traduce nonlinear envelope memory effects.

So, no transfer memoryless nonlinearity located between any two linear filters (in our case $M_s(\omega)$, $M_o(\omega)$ or $H(\omega)$, $O(\omega)$) can be used to represent a RF band-pass nonlinear dynamic system, or even a band-pass memoryless system exhibiting AM-PM conversion. Actually, although the latter behavior could be represented by a polynomial with complex coefficients, that model would still lack dynamic behavior capability for the envelope.

III. NONLINEAR DISTORTION OF BAND-PASS MEMORYLESS AND DYNAMIC SYSTEMS

As seen from (5) to (9) our band-pass memoryless or dynamic nonlinearity produces linear signal components, $y_L(t)$, nonlinear components that are correlated with the signal, $y_N(t)$, and, finally, a form of stochastic nonlinear distortion, $y_S(t)$.

Since $y_N(t)$ is correlated with the input $x(t)$ and the linear path $y_L(t)$, it is, in a certain way, a form of signal, although it arises from a nonlinear process. It is, actually, the responsible for the PA input level induced gain variations described by AM-AM and AM-PM conversion.

When the PA behaves as a band-pass memoryless system subject to a large number of input tones of similar amplitude, the output signal components can be related to the input by a simple linear transfer function which is constant within the bandwidth. Therefore, $y_L(t)+y_N(t)$ will be nothing more than a scaled version of $x(t)$, in which that scaling factor – the nonlinear gain – depends on the averaged operating power: $P_{in}=(1/2)\sum|X_q|^2$.

However, in case of a band-pass dynamic system (where $F(\omega_{LF})$ or $F(\omega_{HF}) \neq 0$) this resulting nonlinear gain will not be a simple scaling factor, but an equivalent linear transfer function that depends on the PA's LF and HF

feedback and, eventually, on the actual distribution of power within the input bandwidth.

As a consequence, and from the point of view of nonlinear gain, $y_N(t)$ should be considered a form of signal perturbation in systems where the actual value of signal amplitude is relevant (like instrumentation and measurement systems) – from now on classified as systems of Type A in this text – or even in any other band-pass dynamic systems that may include memoryless automatic gain control, AGC, (i.e. whose loop bandwidth is much narrow that the signal's envelope) – i.e. systems of Type B. Finally, modern digital wireless systems in which the output is cross-correlated with a template of the expected input, to equalize any eventual linear transfer function, can, to a certain extent, get rid of the dynamic effects pressed into $y_N(t)$, and are thus classified as systems of Type C.

So, for systems of Type A, every output component except $y_L(t)$ should be considered as distortion. CCPR (Co-Channel Power Ratio) proposed in [4], seems to be the adequate figure of merit, as it subtracts from measured output only the components resulting from the small-signal linear processing of the input, $y_L(t)$.

Because systems of Type B include a memoryless AGC loop, they can compensate the output from any static variation of gain, but not from its dynamic changes. Therefore, the appropriate distortion characterization procedure should eliminate $y_L(t)$ from the measured output, along with any $y_N(t)$ component that is related to $y_L(t)$ by a mere constant. This could be reached with a set-up similar to CCPR, where the signal cancellation loop is no longer adjusted for the linear, but for the compressed gain. That is what is done with the CIR (Carrier to Interference Ratio) distortion figure of merit and its measurement set-up adopted in [5].

Finally, in systems of Type C, which can eventually even equalize the signal for dynamic variations, neither CCPR or CIR are appropriate, as they can not eliminate from measured data any nonlinear dynamic variation of gain. Therefore, it seems that for those idealized systems co-channel interference can only be assessed by NPR (Noise Power Ratio). Unfortunately, in band-pass systems presenting strong memory effects it may happen that the distortion characteristics vary significantly with the actual distribution of power within the signal bandwidth. Moreover, there is also experimental and theoretical evidence that even in memoryless systems different signal statistics produce different distortion levels [6]. Hence, in those cases, the NPR test should not be performed with the traditional band-limited white Gaussian noise, but with a sample of the actual input signal the PA is intended to handle. But, even in those cases, it is necessary to guarantee that the NPR notch does not alter significantly

the signal's power spectral density function or time statistics.

IV. ILLUSTRATIVE SIMULATED EXAMPLE

In order to exemplify some of the theoretical derivations, a particular realization of the simplified PA model of Fig. 1, shown in Fig.3, was simulated for CCPR, CIR and NPR. Its circuit values were selected so that it could present significant band-pass dynamic behavior. These predicted results are presented in Fig. 4.

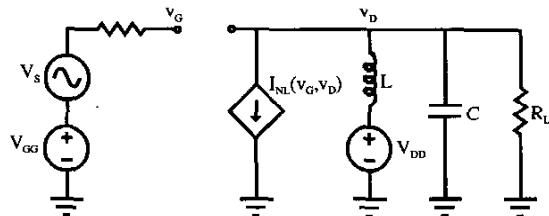


Fig. 3 Simplified PA circuit used for distortion simulations.

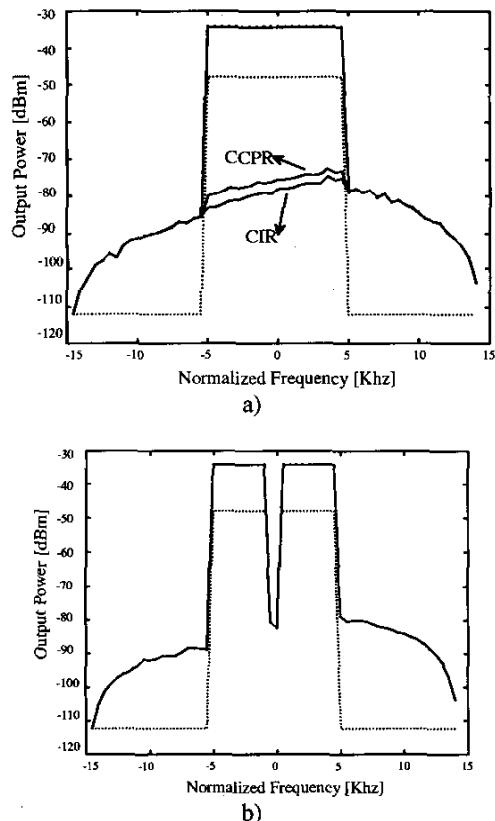


Fig. 4 a) - CCPR and CIR Measurements, and b) - NPR measurements for the simplified band-pass dynamic PA circuit.

The evident asymmetry showed by spectral regrowth in these plots is a clear indication of the presence of envelope memory effects [2]. As a consequence, the distinct non constant co-channel distortion, even with an input and output of flat power distribution over frequency, is illustrative of the increased distortion characterization complexity posed by these band-pass dynamic systems.

V. CONCLUSIONS

A conceptual representation for a band-pass memoryless or dynamic-nonlinear PA was derived, and then used to extract some broad conclusions on the minimum modeling requirements of these sub-systems. It was shown that a typical arrangement using a cascade of a filter followed by a memoryless transfer nonlinearity and then another filter is not able to represent either AM-PM or envelope dynamic behavior.

Finally, this model was also used to derive most important distortion characteristics of those band-pass nonlinear systems, which allowed the discussion of the convenience of some proposed measurement distortion figures of merit and laboratory set-ups.

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